

IB MATH STUDIES SL: IB-Summer Homework

Geometry and Trigonometry

1. A shipping container is a cuboid with dimensions 16 m, $1\frac{3}{4}$ m and $2\frac{2}{3}$ m.

(a) Calculate the **exact** volume of the container. Give your answer as a fraction.

Jim estimates the dimensions of the container as 15 m, 2 m and 3 m and uses these to estimate the volume of the container.

(b) Calculate the percentage error in Jim's estimated volume of the container.

(Total 6 marks)

2. The base of a prism is a **regular hexagon**. The centre of the hexagon is O and the length of OA is 15 cm.

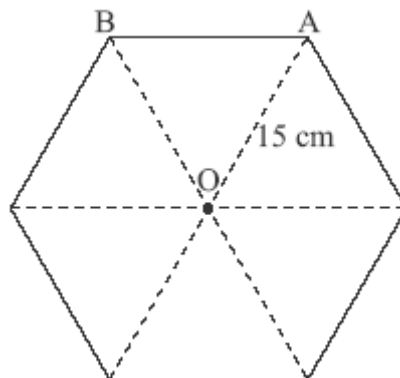


diagram not to scale

(a) Write down the size of angle AOB.

(b) Find the area of the triangle AOB.

The height of the prism is 20 cm.

(c) Find the volume of the prism.

(Total 6 marks)

3. The area of a circle is equal to 8 cm^2 .

(a) Find the radius of the circle.

This circle is the base of a **solid** cylinder of height 25 cm.

(b) Write down the volume of the **solid** cylinder.

(c) Find the **total** surface area of the **solid** cylinder.

(Total 6 marks)

4. A field is 91.4 m long and 68.5 m wide.

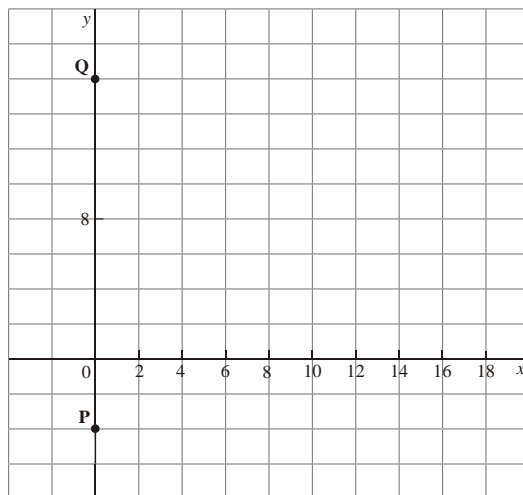
(a) Calculate the area of the field in m^2 .

(b) Calculate the area of the field in cm^2 .

(c) Express your answer to (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(Total 6 marks)

5. Points $P(0, -4)$, $Q(0, 16)$ are shown on the diagram.



(a) Plot the point $R(11, 16)$.

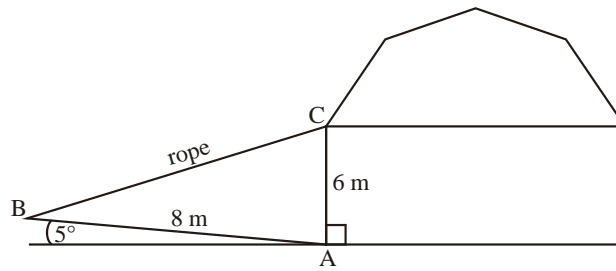
(b) Calculate angle \hat{QPR} .

M is a point on the line PR . M is 9 units from P .

(c) Calculate the area of triangle PQM .

(Total 6 marks)

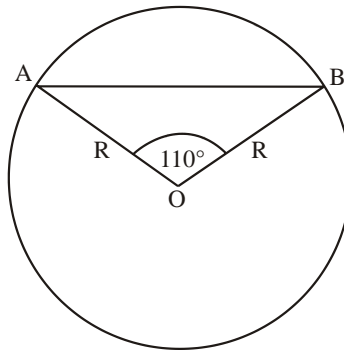
6. The following diagram shows the side view of a tent. The side of the tent AC is 6 m high. The ground AB slopes upwards from the bottom of the tent at point A, at an angle of 5° from the horizontal. The tent is attached to the ground by a rope at point B, a distance of 8 m from its base.



- Calculate the angle BAC.
- Calculate the length of the rope, BC.
- Calculate the angle CBA that the rope makes with the sloping ground.

(Total 8 marks)

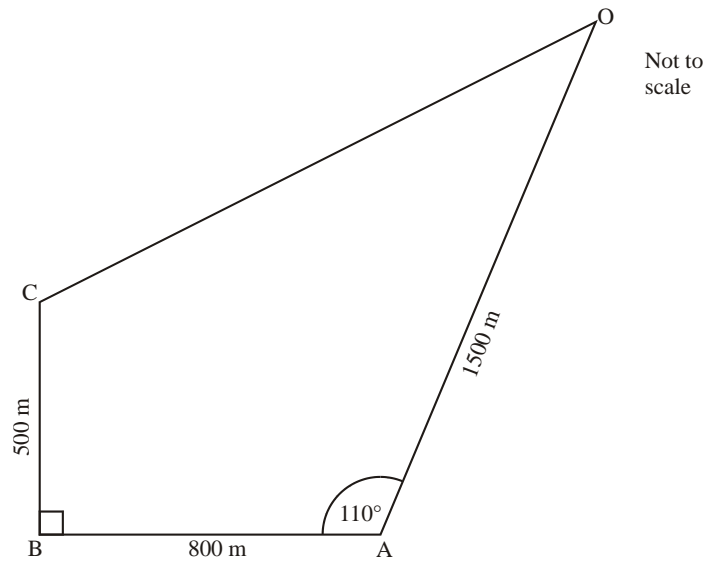
7. The diagram shows a circle of radius R and centre O . A triangle AOB is drawn inside the circle. The vertices of the triangle are at the centre, O , and at two points A and B on the circumference. Angle \hat{AOB} is 110° .



- Given that the area of the circle is $36\pi \text{ cm}^2$, calculate the length of the radius R .
- Calculate the length AB .
- Write down the side length L of a square which has the same area as the given circle.

(Total 6 marks)

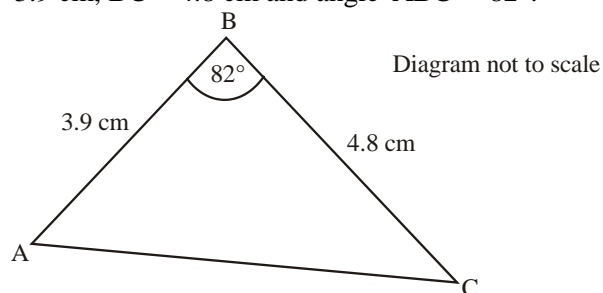
8. A cross-country running course is given in the diagram below. Runners start and finish at point O.



- (a) Show that the distance CA is 943 m correct to 3 s.f. (2)
- (b) Show that angle BCA is 58.0° correct to 3 s.f. (2)
- (c) (i) Calculate the angle CAO. (5)
- (ii) Calculate the distance CO. (4)
- (d) Calculate the area enclosed by the course OABC. (4)
- (e) Gonzales runs at a speed of 4 m s^{-1} . Calculate the time, in minutes, taken for him to complete the course. (3)

(Total 16 marks)

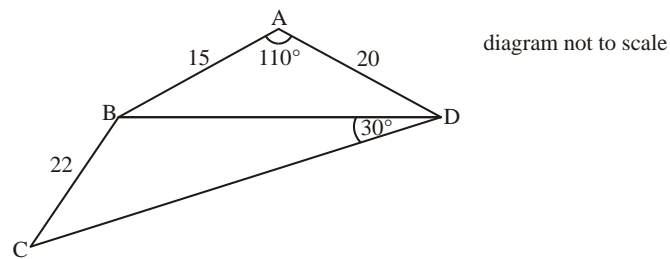
9. In triangle ABC, $AB = 3.9 \text{ cm}$, $BC = 4.8 \text{ cm}$ and angle $\hat{A}BC = 82^\circ$.



- (a) Calculate the length of AC.
- (b) Calculate the size of angle $\hat{A}CB$.

(Total 6 marks)

10. The diagram below shows a field ABCD with a fence BD crossing it. $AB = 15\text{m}$, $AD = 20\text{m}$ and angle $\hat{BAD} = 110^\circ$. $BC = 22\text{m}$ and angle $\hat{BDC} = 30^\circ$.



- (a) Calculate the length of BD. (3)
- (b) Calculate the size of angle \hat{BCD} . (3)

One student gave the answer to (a) “correct to 1 significant figure” and used this answer to calculate the size of angle \hat{BCD} .

- (c) Write down the length of BD correct to 1 significant figure. (1)
- (d) Find the size of angle \hat{BCD} that the student calculated, **giving your answer correct to 1 decimal place.** (2)
- (e) Hence find the percentage error in his answer for angle \hat{BCD} . (3)

(Total 12 marks)

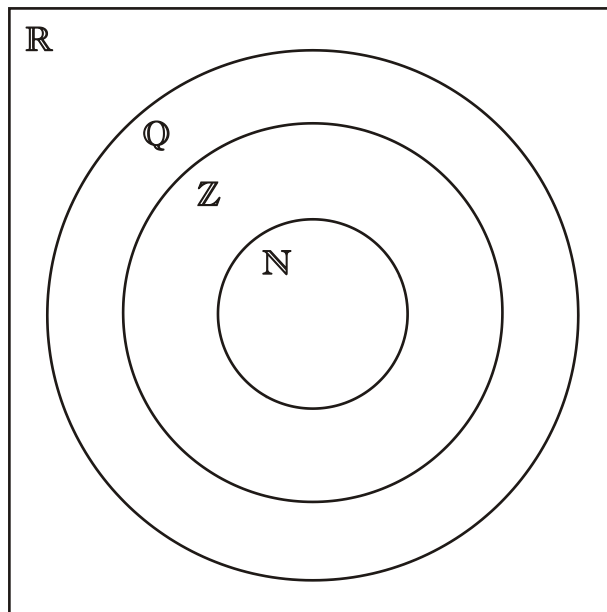
Sets, Venn Diagrams, Scientific Notation, and Approximation

11. The Venn diagram below shows the universal set of real numbers \mathbb{R} and some of its important subsets:

\mathbb{Q} : the rational numbers,
 \mathbb{Z} : the integers,
 \mathbb{N} : the natural numbers.

Write the following numbers in the correct position in the diagram.

$$-1, 1, \pi, \frac{7}{16}, 3.333\overline{3}, \sqrt{3}.$$



(Total 6 marks)

12. Let $U = \{-4, -\frac{2}{3}, 1, \pi, 13, 26.7, 69, 10^{33}\}$.

A is the set of all the integers in U .

B is the set of all the rational numbers in U .

- (a) List all the prime numbers contained in U . (2)
- (b) List all the members of A . (2)
- (c) List all the members of B . (2)
- (d) List all the members of the set $A \cap B$. (2)

(Total 8 marks)

13. The universal set U is the set of integers from 1 to 20 inclusive.

A and B are subsets of U where:

A is the set of even numbers between 7 and 17.

B is the set of multiples of 3.

List the elements of the following sets:

- (a) A ; (1)
- (b) B ; (1)
- (c) $A \cup B$; (2)
- (d) $A \cap B'$. (2)

(Total 6 marks)

14. (a) Given $x = 2.6 \times 10^4$ and $y = 5.0 \times 10^{-8}$, calculate the value of $w = x \times y$. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. (4)

(b) Which **two** of the following statements about the nature of x , y and w above are **incorrect**? (4)

(i) $x \in \mathbb{N}$

(ii) $y \in \mathbb{Z}$

(iii) $y \in \mathbb{Q}$

(iv) $w < y$

(v) $x + y \in \mathbb{R}$

(vi) $\frac{1}{w} < x$

(Total 8 marks)

15. A rectangle has length 2.6×10^4 and width 1.9×10^4 . Find each of the following, giving your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(a) The area of the rectangle (4)

(a) The perimeter of the rectangle (4)

(Total 8 marks)

16. Anthony uses the formula $p = \frac{27q}{r+s}$ to calculate the value of p .

Suppose that correct to two decimal places, $q = 0.89$, $r = 1.87$ and $s = 7.22$.

(a) He estimates the value **without using a calculator**.

(i) Write down the numbers Anthony could use in the formula to estimate the value of p .

(1)

(ii) Work out the estimate for the value of p that your numbers would give.

(1)

(b) A calculator is to be used to work out the actual value of p .

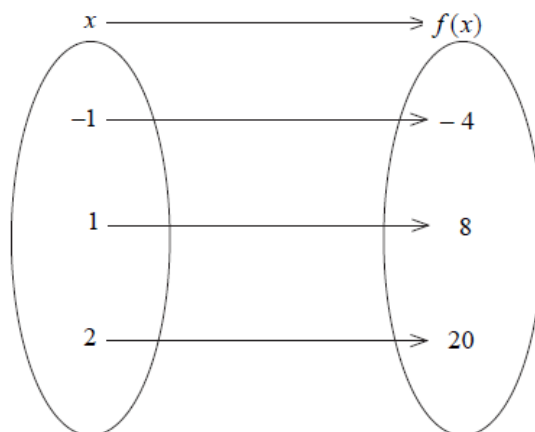
To what degree of accuracy would you give your calculator answer? Give a reason for your answer.

(2)

(Total 4 marks)

Linear and Simultaneous Equations, Linear Relations, and Functions

17. A quadratic function, $f(x) = ax^2 + bx$, is represented by the mapping diagram below.



(a) Use the mapping diagram to write down **two** equations in terms of a and b .

(2)

(b) Find the value of

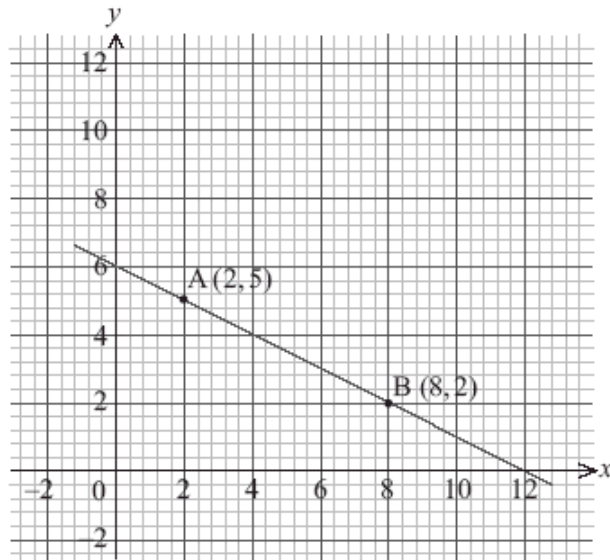
(i) a ;

(ii) b .

(2)

(Total 4 marks)

18. A and B are points on a straight line as shown on the graph below.



(a) Write down the y-intercept of the line AB.

(1)

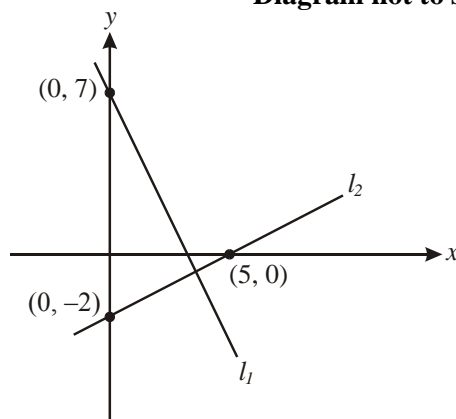
(b) Calculate the gradient of the line AB.

(2)

(Total 3 marks)

19. The following diagram shows the lines l_1 and l_2 , which are perpendicular to each other.

Diagram not to scale



(a) Calculate the gradient of line l_1 .

(b) Write the equation of line l_1 in the form $ax + by + d = 0$ where a , b and d are integers, and $a > 0$.

(Total 8 marks)

20. The diagram shows triangle ABC. Point C has coordinates (4, 7) and the equation of the line AB is $x + 2y = 8$.

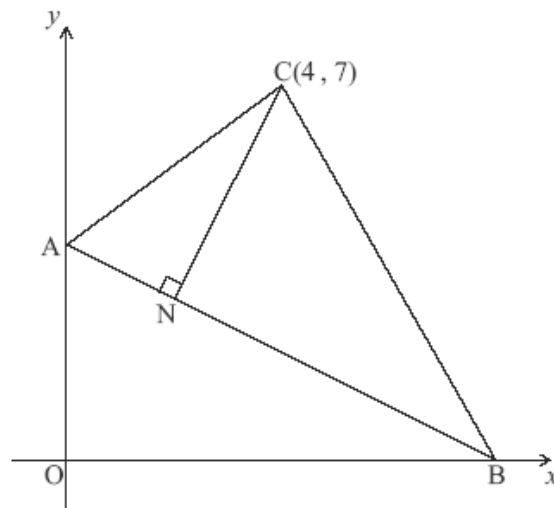


diagram not to scale

- (a) Find the coordinates of
- (i) A;
 - (ii) B.
- (2)
- (b) Show that the distance between A and B is 8.94 correct to 3 significant figures.
- (2)
- N lies on the line AB. The line CN is perpendicular to the line AB.
- (c) Find
- (i) the gradient of CN ;
 - (ii) the equation of CN.
- (5)
- (d) Calculate the coordinates of N.
- (3)
- (Total 12 marks)**

21. $P(4, 1)$ and $Q(0, -5)$ are points on the coordinate plane.

- (a) Determine the
- (i) coordinates of M , the midpoint of P and Q ;
 - (ii) gradient of the line drawn through P and Q ;
 - (iii) gradient of the line drawn through M , perpendicular to PQ .

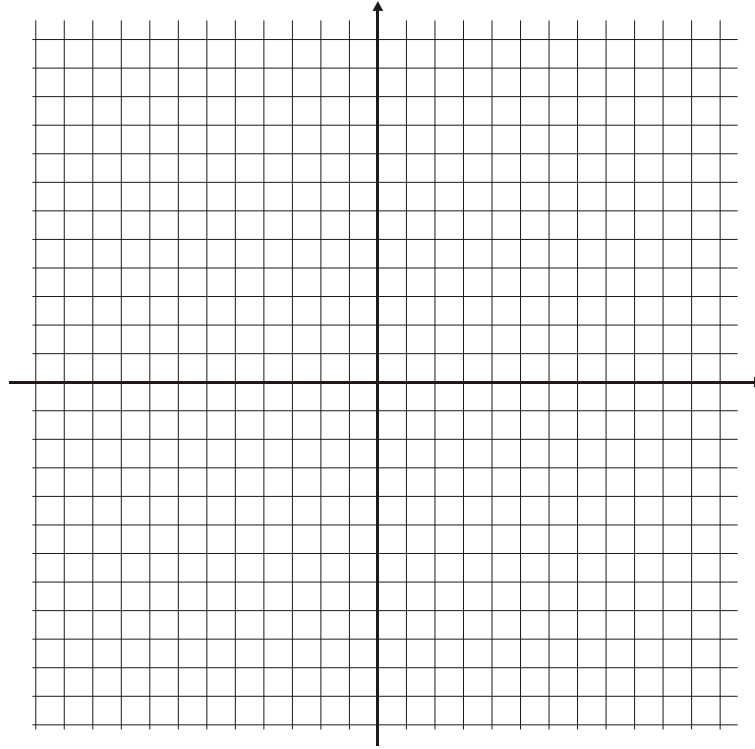
The perpendicular line drawn through M meets the y -axis at $R(0, k)$.

- (b) Find k .

(Total 6 marks)

22. Three points A (1, 3), B (4, 10) and C (7, -1) are joined to form a triangle. The mid-point of AB is D and the mid-point of AC is E.

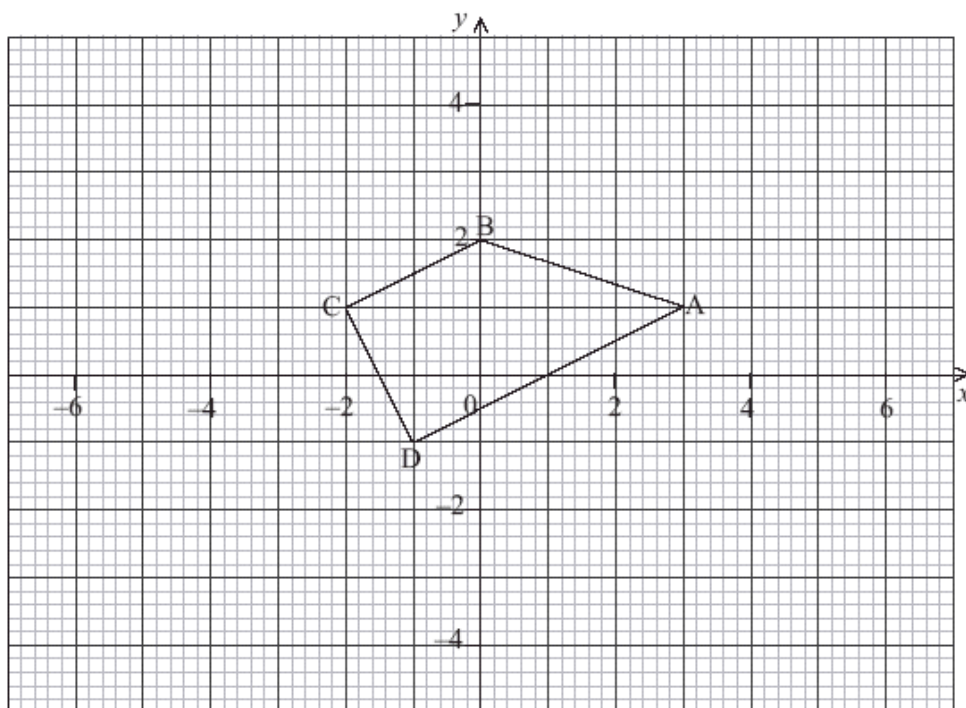
(a) Plot the points A, B, C, on the grid.



(b) Find the distance DE.

(Total 6 marks)

23. The vertices of quadrilateral ABCD as shown in the diagram are A (3, 1), B (0, 2), C (-2, 1) and D (-1, -1).



- (a) Calculate the gradient of line CD. (2)
- (b) Show that line AD is perpendicular to line CD. (2)
- (c) Find the equation of line CD. Give your answer in the form $ax + by = c$ where $a, b, c \in \mathbb{Z}$. (3)

Lines AB and CD intersect at point E. The equation of line AB is $x + 3y = 6$.

- (d) Find the coordinates of E. (2)
- (e) Find the distance between A and D. (2)
- (Total 11 marks)**

24. A is the point (2, 3), and B is the point (4, 9).

- (a) Find the gradient of the line segment [AB].
- (b) Find the gradient of a line perpendicular to the line segment [AB].
- (c) The line $2x + by - 12 = 0$ is perpendicular to the line segment [AB]. What is the value of b ?

(Total 4 marks)

25. The diagram below shows the line PQ, whose equation is $x + 2y = 12$. The line intercepts the axes at P and Q respectively.

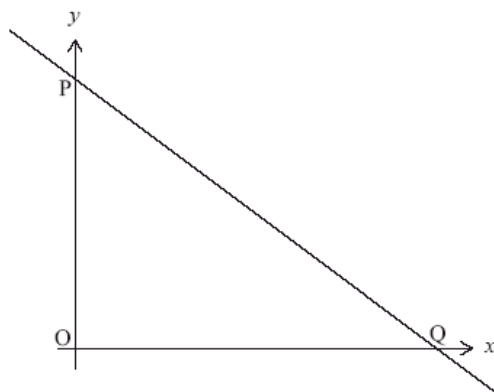


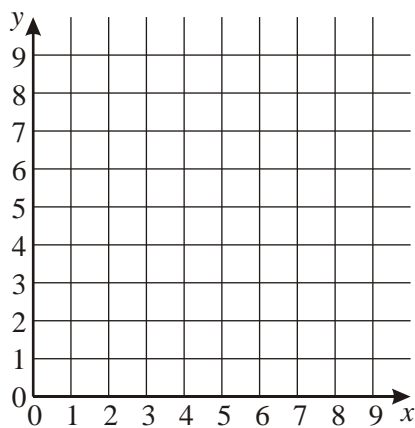
diagram not to scale

- (a) Find the coordinates of P and of Q. (3)
- (b) A second line with equation $x - y = 3$ intersects the line PQ at the point A. Find the coordinates of A. (3)

(Total 6 marks)

26. The equation of a line l_1 is $y = \frac{1}{2}x$.

- (a) On the grid, draw and label the line l_1 .



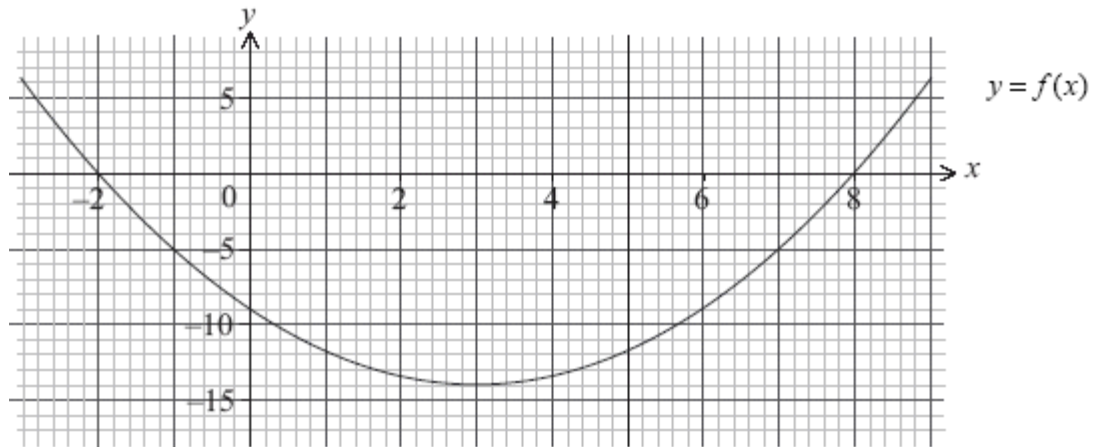
The line l_2 has the same gradient as l_1 , but crosses the y-axis at 3.

- (b) What is the geometric relationship between l_1 and l_2 ?
- (c) Write down the equation of l_2 .
- (d) On the same grid as in part (a), draw the line l_2 .

(Total 4 marks)

Quadratic Equations and Functions

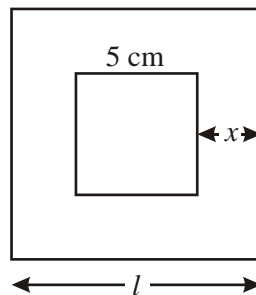
27. The graph of a quadratic function $y = f(x)$ is given below.



- (a) Write down the equation of the axis of symmetry. (2)
- (b) Write down the coordinates of the minimum point. (2)
- (c) Write down the range of $f(x)$. (2)

(Total 6 marks)

28. A picture is in the shape of a square of side 5 cm. It is surrounded by a wooden frame of width x cm, as shown in the diagram below.



The length of the wooden frame is l cm, and the area of the wooden frame is A cm².

- (a) Write an expression for the length l in terms of x . (1)
- (b) Write an expression for the area A in terms of x . (2)
- (c) If the area of the frame is 24 cm², find the value of x . (4)

(Total 7 marks)

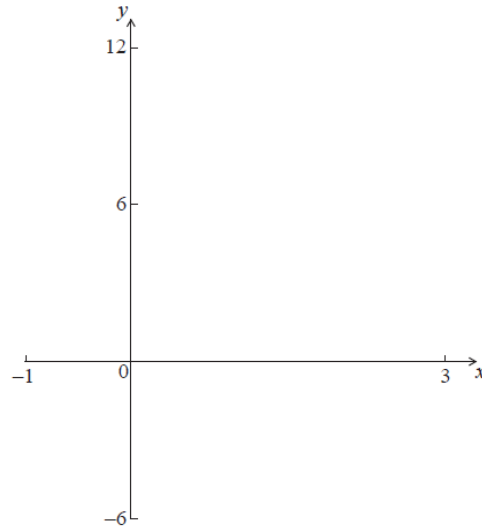
29. The x -coordinate of the minimum point of the quadratic function $f(x) = 2x^2 + kx + 4$ is $x = 1.25$.

(a) (i) Find the value of k .

(ii) Calculate the y -coordinate of this minimum point.

(4)

(b) Sketch the graph of $y = f(x)$ for the domain $-1 \leq x \leq 3$.



(2)

(Total 6 marks)

30. A rectangle has dimensions $(5 + 2x)$ metres and $(7 - 2x)$ metres.

(a) Show that the area, A , of the rectangle can be written as $A = 35 + 4x - 4x^2$.

(1)

(b) The following is the table of values for the function $A = 35 + 4x - 4x^2$.

x	-3	-2	-1	0	1	2	3	4
A	-13	p	27	35	q	r	11	s

(i) Calculate the values of p , q , r and s .

(ii) On graph paper, using a scale of 1 cm for 1 unit on the x -axis and 1 cm for 5 units on the A -axis, plot the points from your table and join them up to form a smooth curve.

(6)

(c) Answer the following, using your graph or otherwise.

(i) Write down the equation of the axis of symmetry of the curve,

(ii) Find one value of x for a rectangle whose area is 27 m^2 .

(iii) Using this value of x , write down the dimensions of the rectangle.

(4)

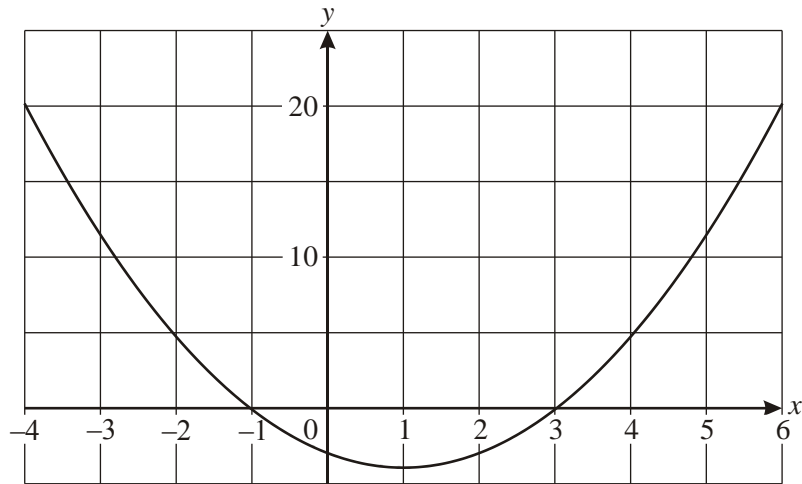
(d) (i) On the same graph, draw the line with equation $A = 5x + 30$.

(ii) Hence or otherwise, solve the equation $4x^2 + x - 5 = 0$.

(3)

(Total 14 marks)

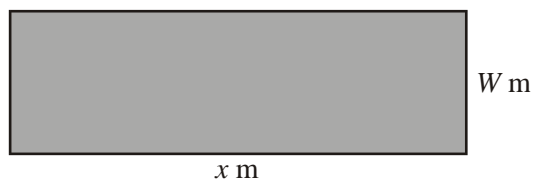
31. The graph of $y = x^2 - 2x - 3$ is shown on the axes below.



- (a) Draw the graph of $y = 5$ on the same axes.
- (b) Use your graph to find:
- the values of x when $x^2 - 2x - 3 = 5$
 - the value of x that gives the minimum value of $x^2 - 2x - 3$

(Total 4 marks)

32. The perimeter of this rectangular field is 220 m. One side is x m as shown.



- Express the width (W) in terms of x .
- Write an expression, in terms of x only, for the area of the field.
- If the length (x) is 70 m, find the area.

(Total 4 marks)

33. The graph of a quadratic function $f(x)$ intersects the horizontal axis at $(1, 0)$ and the equation of the axis of symmetry is $x = -1$.

- (a) Write down the x -coordinate of the other point where the graph of $y = f(x)$ intersects the horizontal axis.
- (b) $y = f(x)$ reaches its maximum value at $y = 5$.
 - (i) Write down the value of $f(-1)$.
 - (ii) Find the range of the function $y = f(x)$.

(Total 6 marks)

34. (a) Factorize $3x^2 + 13x - 10$.

(2)

(b) Solve the equation $3x^2 + 13x - 10 = 0$.

(2)

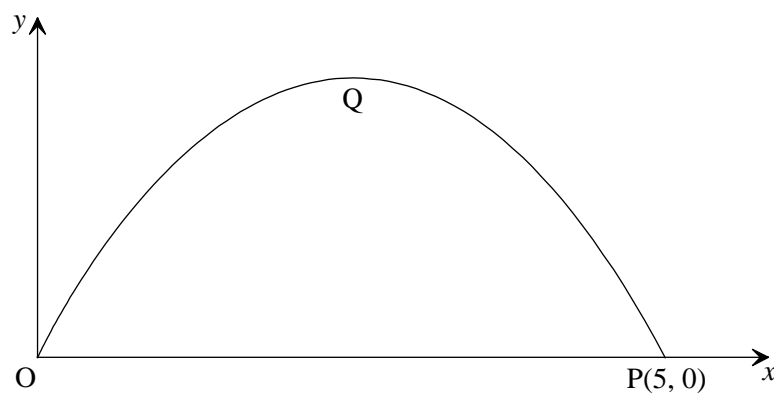
Consider a function $f(x) = 3x^2 + 13x - 10$.

(c) Calculate the minimum value of this function.

(2)

(Total 6 marks)

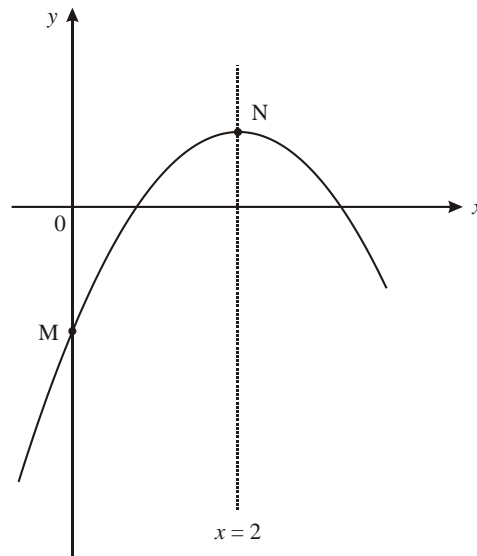
35. The diagram below shows the graph of $y = c + kx - x^2$, where k and c are constants.



- (a) Find the values of k and c .
- (b) Find the coordinates of Q, the highest point on the graph.

(Total 8 marks)

36. The diagram below shows part of the graph of $y = ax^2 + 4x - 3$. The line $x = 2$ is the axis of symmetry. M and N are points on the curve, as shown.



- (a) Find the value of a .
- (b) Find the coordinates of
- M;
 - N.

(Total 4 marks)

37. The function $Q(t) = 0.003t^2 - 0.625t + 25$ represents the amount of energy in a battery after t minutes of use.

- State the amount of energy held by the battery immediately before it was used.
- Calculate the amount of energy available after 20 minutes.
- Calculate the number of minutes it takes for the energy to reach zero.

(Total 4 marks)

38. The profit (P) in Swiss Francs made by three students selling homemade lemonade is modeled by the function

$$P = -\frac{1}{20}x^2 + 5x - 30$$

where x is the number of glasses of lemonade sold.

- (a) **Copy** and complete the table below

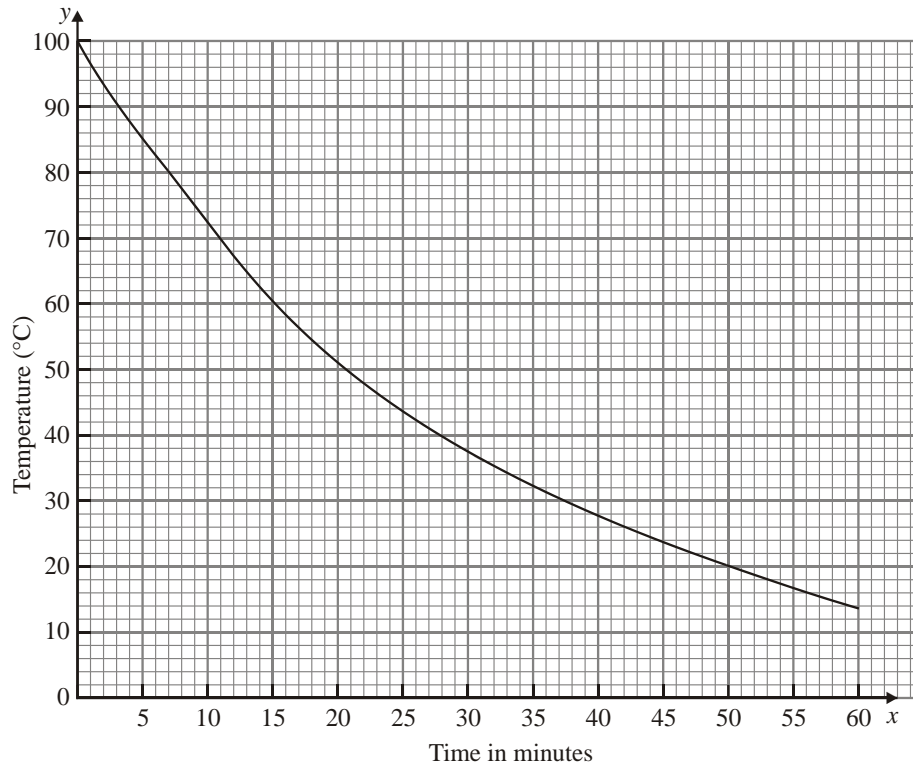
x	0	10	20	30	40	50	60	70	80	90
P		15			90			75	50	

- (b) On graph paper draw axes for x and P , placing x on the horizontal axis and P on the vertical axis. Use suitable scales. Draw the graph of P against x by plotting the points. Label your graph.
- (c) **Use your graph** to find
- the maximum possible profit;
 - the number of glasses that need to be sold to make the maximum profit;
 - the number of glasses that need to be sold to make a profit of 80 Swiss Francs;
 - the amount of money initially invested by the three students.
- (d) The three students Baljeet, Jane and Fiona share the profits in the ratio of 1:2:3 respectively. If they sold 40 glasses of lemonade, calculate Fiona's share of the profits.

(Total 15 marks)

Exponential Equations

39. The graph below shows the temperature of a liquid as it is cooling.



- (a) Write down the temperature after 5 minutes.
- (b) After how many minutes is the temperature 50°C ?

The equation of the graph for all positive x can be written in the form $y = 100(5^{-0.02x})$.

- (c) Calculate the temperature after 80 minutes.
- (d) Write down the equation of the asymptote to the curve.

(Total 8 marks)

Probability

40. A fair six-sided die has the numbers 1, 2, 3, 4, 5, 6 written on its faces. A fair four-sided die has the numbers 1, 2, 3, and 4 written on its faces. The two dice are rolled.

- (a) Find the probability that the two dice show the same number. (2)
- (b) Find the probability that the difference between the two numbers shown on the dice is 1. (2)
- (c) Find the probability that the number shown on the four-sided die is greater than the number shown on the six-sided die, given that the difference between the two numbers is 1. (2)

(Total 6 marks)

41. When Andy plays tennis, 65% of his first serves go into the correct area of the court.

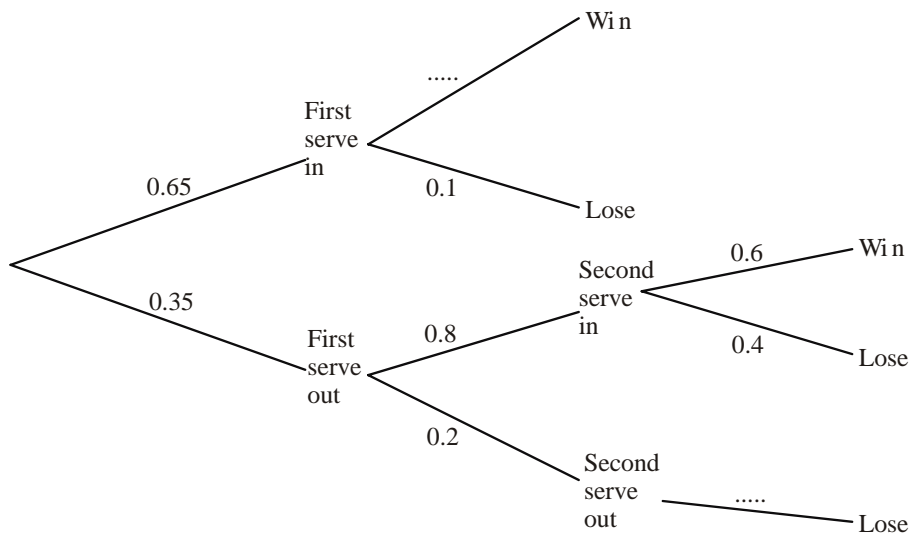
If the first serve goes into the correct area, his chance of winning the point is 90%.

If his first serve does not go into the correct area, Andy is allowed a second serve and, of these, 80% go into the correct area.

If the second serve goes into the correct area, his chance of winning the point is 60%.

If neither serve goes into the correct area, Andy loses the point.

- (a) Complete the tree diagram below.



(2)

- (b) Find the probability that Andy loses the point.

(4)

(Total 6 marks)

42. For events A and B , the probabilities are $P(A) = \frac{4}{13}$ and $P(B) = \frac{5}{13}$.

(a) If events A and B are mutually exclusive, write down the value of $P(A \cap B)$. (1)

(b) If events A and B are independent, find the value of $P(A \cap B)$. (2)

(c) If $P(A \cup B) = \frac{7}{13}$, find the value of $P(A \cap B)$. (3)

(Total 6 marks)

43. A class consists of students studying Spanish or French or both. Fifteen students study Spanish and twelve study French.

The probability that a student studies French given that she studies Spanish is $\frac{7}{15}$.

(a) Draw a Venn diagram to illustrate this information. (3)

(b) Find the probability that a student studies Spanish given that she studies one language only. (3)

(Total 6 marks)

44. A survey of 100 families was carried out, asking about the pets they own. The results are given below.

56 owned dogs (S)
38 owned cats (Q)
22 owned birds (R)
16 owned dogs and cats, but not birds
8 owned birds and cats, but not dogs
3 owned dogs and birds, but not cats
4 owned all three types of pets

(a) Draw a Venn diagram to represent this information. (5)

(b) Find the number of families who own no pets. (2)

(c) Find the percentage of families that own exactly one pet. (3)

(d) A family is chosen at random. Find the probability that they own a cat, given that they own a bird. (2)

(Total 12 marks)

45. A survey of 400 people is carried out by a market research organization in two different cities, Buenos Aires and Montevideo. The people are asked which brand of cereal they prefer out of Chocos, Zucos or Fruti. The table below summarizes their responses.

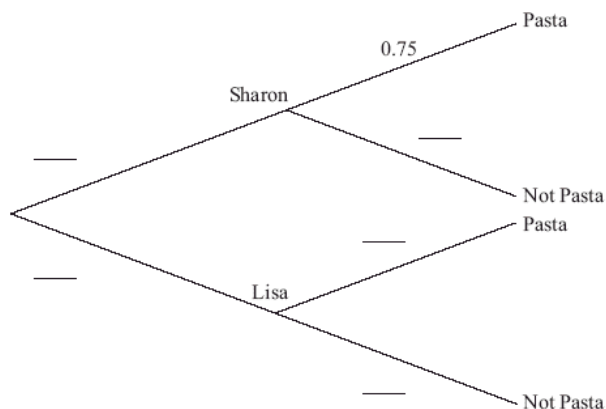
	Chocos	Zucos	Fruti	Total
Buenos Aires	43	85	62	190
Montevideo	57	35	118	210
Total	100	120	180	400

- (a) One person is chosen at random from those surveyed. Find the probability that this person
- does not prefer Zucos;
 - prefers Chocos, given that they live in Montevideo.
- (4)
- (b) Two people are chosen at random from those surveyed. Find the probability that they both prefer Fruti.
- (3)

(Total 7 marks)

46. Sharon and Lisa share a flat. Sharon cooks dinner three nights out of ten. If Sharon does not cook dinner, then Lisa does. If Sharon cooks dinner the probability that they have pasta is 0.75. If Lisa cooks dinner the probability that they have pasta is 0.12.

- (a) **Copy and complete** the tree diagram to represent this information.



- (3)
- (b) Find the probability that Lisa cooks dinner and they do not have pasta.
- (2)
- (c) Find the probability that they do not have pasta.
- (3)
- (d) Given that they do not have pasta, find the probability that Lisa cooked dinner.
- (3)

(Total 11 marks)